Low-energy neutrino-photon inelastic interactions

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Abstract

The computation of the polarized amplitudes and cross section of the processes $\gamma\nu \rightarrow \gamma\gamma\nu$, $\gamma\gamma \rightarrow \gamma\nu\bar{\nu}$ and $\nu\bar{\nu} \rightarrow \gamma\gamma\gamma$ is described. We used an effective lagrangian approach for energies below the threshold for e^+e^- pair production and the complete computation at higher energies for application in supernova dynamics. Leading contributions of physics beyond the SM are also commented.

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I. INTRODUCTION

Low-energy neutrino–photon interactions are of potential interest in astrophysics, since they could affect the mechanism of stars to lose energy and hence the study of the stellar evolution, in particular, supernova dynamics.

However the cross sections of the $2 \to 2$ processes ($\gamma\gamma \to \nu\bar{\nu}$, $\gamma\nu \to \gamma\nu$ and $\nu\bar{\nu} \to \gamma\gamma$) are too small. The reason for their strong suppression is the prohibition of the coupling of two photons to a $J = 1$ state of any parity. This is simply Yang's theorem [1]. As a consequence the amplitudes of the $2 \to 2$ processes are zero to order G_F. For instance [2],

$$A^{\lambda\lambda'}(\gamma\nu \to \gamma\nu) = \frac{1}{2\pi} \frac{g^2 \alpha}{M_W^4} \left(1 + \frac{4}{3} \log \frac{M_W^2}{m_e^2}\right) \cos\frac{\theta}{2} f^{\lambda\lambda'}, \tag{1}$$

where λ, λ' are the photon helicities and $f^{\lambda\lambda'}$ is a certain function of s and t. The important point to notice here is that the scale of the process is given by M_W, which strongly suppresses it at low energies. However there is a way to bypass this suppression: if one couples three photons instead of two the theorem does not apply anymore. This implies on the one hand, that we will pay an extra α in the cross section. But, on the other, that it is possible that the scale of the process is no longer M_W but some other light scale that could give a large enhancement.

The processes will then be

$$\gamma\gamma \to \nu\bar{\nu}\gamma, \qquad \gamma\nu \to \gamma\gamma\nu, \qquad \text{and} \qquad \nu\bar{\nu} \to \gamma\gamma\gamma \tag{2}$$

The first may affect the stellar energy loss mechanism and the other two may reduce the mean free path of a neutrino inside a supernova core and in principle they could act as a cut off of high energy photons or neutrinos by the background. The only existing computation in the literature of one of these complicated processes disagrees [3] with a recent computation by Dicus and Repko [4]. Our aim in [5] was to settle down this disagreement and give an explicit derivation of this effective lagrangian that could be useful for applications in a different context.

Numerically, it was found that the cross section of the inelastic 5-leg processes are between **9 and 13 orders of magnitude** larger than the $2 \to 2$ corresponding processes for center of mass energies ω between 0.2 and 2 times the mass of the electron m_e.

However if one needs to go to higher energies, for instance to energies above the threshold for e^+e^- pair production, an exact calculation of the processes in Eq. (2) was important so as to definitively assess their role in astrophysics and the range of validity of the effective theory.

In this talk we will comment on both approaches: the effective lagrangian and the direct computation in the SM. We will also briefly comment on how physics beyond the SM could affect them.

II. EFFECTIVE LAGRANGIAN APPROACH

Three main observations will allow us to obtain the leading contribution at low energies (of the order of m_e) to the cross section of the processes in Eq. (2):

(i) First, the lack of the $1/M_W^4$ suppression in the 5-leg processes (Yang's theorem does not apply there) suggest that we concentrate on those diagrams with a lighter particle inside the loop, for instance m_e.

(ii) Second, the processes in Eq. (2) are finite, no counterterm is needed. This observation helps in the search for the leading diagrams by defining a hierarchy of diagrams at low energies. Giving a certain topology it becomes easier to find its large m_e limit. These two observations imply that the leading diagrams contributing to these processes are those given in Fig. 1. Any other diagram or topology will include other particles inside the loop, different from the electron, and they will be automatically suppressed by inverse powers of its mass.
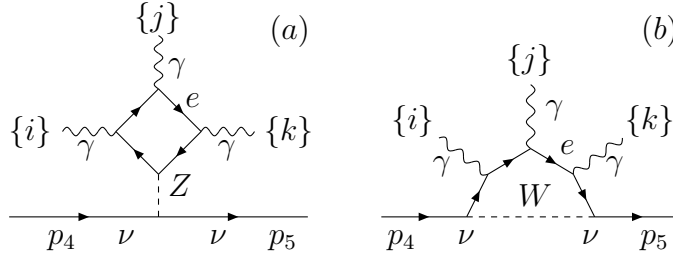

Figure 1. SM leading diagrams contributing to five-leg photon–neutrino processes: a) Type A diagrams b) Type B diagrams

(iii) So far we have found the leading diagrams. A third observation will allow us to obtain its low-energy expansion in an elegant way: the subset of diagrams depicted in Fig. 1 resembles a photon–photon scattering process (see Fig. 2), one of the photons being off-shell.
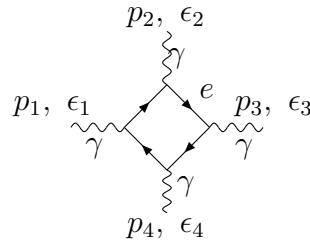

Figure. 2: Four-photon interaction: Type C diagrams

Before expressing this "resemblance" in mathematical terms we should clarify why it is interesting to make this link. The reason is as follows: we know that the SM is able to describe both the photon–photon scattering process and our inelastic photon–neutrino process which we want to compute. Moreover, we know that the low-energy limit of the photon–photon scattering process at one loop is given by the Euler–Heisenberg lagrangian [6]. Hence if we are able to find a connection between the diagrams of the two processes, we will automatically be able to establish a link of the effective lagrangian of our process in terms of the Euler–Heisenberg lagrangian of the photon–photon scattering without having to compute a single one-loop diagram.

In order to establish the link between those two processes, we should be able to answer two questions:

(a) Can type B diagrams be reduced to type A diagrams?

(b) Is it possible to relate type A diagrams, which contain a vertex Zee with an axial part $ig\gamma_\mu(v_e + a_e\gamma_5)/(2c_\theta)$, with type C diagrams whose corresponding vertex $igs_\theta\gamma_\mu$ does not?

It was shown in [5] that it is possible to give a positive answer to both questions. The keypoints of the proof are two: on the one hand, the gauge boson propagators should be expanded in the large M_W, M_Z limit. This is justified since we are working at very low energies. On the other hand, we should find out the combination of the amplitudes of type A diagrams with different polarization indices in such a way that the axial part cancels. Indeed, Gell-Mann gave an indirect proof in [7], using charge-conjugation arguments, that such combinations should exist.

In conclusion, at leading order in $1/M_W^2$, it was found in [5] that the following set of four diagrams (from a total of 12) of type A and type B diagrams is proportional to the same integral, called L_1 (see [5] for definitions):

$$A_{123}^{\alpha\beta\gamma} + A_{321}^{\alpha\beta\gamma} + B_{123}^{\alpha\beta\gamma} + B_{321}^{\alpha\beta\gamma} = -\frac{g^5 s_W^3}{2}(1 + v_e)\Gamma_\mu \frac{1}{\Delta_Z c_W^2}L_1^{\mu\alpha\beta\gamma}. \tag{3}$$

Similar results are obtained for the other two groups of four diagrams, the only difference being a trivial change of momenta and indices inside $L_1^{\mu\alpha\beta\gamma}$.

At this point, the correspondence with the four-photon scattering is evident. In fact, by fixing the fourth photon leg and calling $C_{123}^{\alpha\beta\gamma}$ the corresponding amplitude in Fig. 2, one finds a contribution proportional to the same integral L_1:

$$C_{123}^{\alpha\beta\gamma} + C_{321}^{\alpha\beta\gamma} = 2g^4 s_W^4 \epsilon_\mu(\vec{P_4}, \lambda_4)L_1^{\mu\alpha\beta\gamma}, \tag{4}$$

and similar results for the two remaining combinations $C_{132}^{\alpha\beta\gamma} + C_{231}^{\alpha\beta\gamma}$ and $C_{213}^{\alpha\beta\gamma} + C_{312}^{\alpha\beta\gamma}$.

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Up to now, we have proved that both processes are governed by the same integral, so both should have the same momentum dependence. Moreover, we have said that at low energies photon–photon scattering is governed by the Euler–Heisenberg lagrangian [6]. This automatically tells us which are the only operators that can be generated at low energies. We have no freedom either in the structure nor in the relative coefficients between the operators. Equations (3) and (4) suggest to define a new gauge field up to the neutrino current $\tilde{A}_\nu \equiv \bar{\psi}\gamma_\nu(1 - \gamma_5)\psi = 2\Gamma_\nu$, with field strength $\tilde{F}_{\mu\nu}$. Indeed this definition is not unique, so we should add a constant in front of the effective lagrangian that still has to be fixed. The lagrangian then reads

$$\mathcal{L}_{\text{eff}} = \frac{C}{180}\left[5\left(\tilde{F}_{\mu\nu}F^{\mu\nu}\right)\left(F_{\lambda\rho}F^{\lambda\rho}\right) - 14\tilde{F}_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu}\right]. \tag{5}$$

In order to fix this constant, we can use the following ratio of amplitudes

$$\lim_{\text{large } m_e} \frac{\mathcal{A}_{4\gamma}^{\text{SM}}}{\mathcal{A}_P^{\text{SM}}} = \frac{\mathcal{A}_{4\gamma}^{\text{eff}}}{\mathcal{A}_P^{\text{eff}}}, \tag{6}$$

where P is our process and 4γ is the photon scattering. Then we obtain [5]

$$C = \frac{g^5 s_W^3(1 + v_e)}{32\pi^2 m_e^4 M_W^2} = \frac{2G_F \alpha^{3/2}(1 + v_e)}{\sqrt{2}\pi m_e^4}. \tag{7}$$

At this point some remarks are in order. Notice that the suppression factor for the 5-leg processes is $1/M_W^2$, to be compared with the $1/M_W^4$ suppression of the processes with one photon less. Moreover the scale is not given by M_W but m_e, giving rise to an important enhancement. Finally, once we have established the dictionary that translates between the two processes, $C \to \frac{\alpha^2}{m_e^4}$ and $\Gamma_\mu \to 2\epsilon_\mu(\vec{P_4}, \lambda_4)$, one can check each polarization amplitude with the corresponding one of the photon–photon scattering after putting all the photon legs on-shell. This provides a non-trivial check of our computation.

In [5] we obtained, using this effective lagrangian, the eight polarized differential cross sections for the processes $\gamma\nu \to \gamma\nu\gamma$ and $\gamma\gamma \to \nu\bar{\nu}\gamma$. Once added we compared our result with the unpolarized differential cross section and the total cross section given in [4]. We found perfect agreement and hence disagreement with the ones obtained by Hieu and Shabalin [3].

III. DIRECT COMPUTATION IN THE SM AND BEYOND

For energies above the e^+e^- threshold it is required to perform a direct computation in the SM [8,9]. In order to compute the processes in Eq. (2) directly in the SM we need to face a multileg computation. The traditional methods, such as tensorial decomposition [10], suffer from large numerical instabilities due to the large proliferation of terms and, in particular, due to the appearance of gram determinants $\Delta = \det k_i k_j$ that vanish in collinear regions

of the phase space where the cross section is, indeed, well defined. We have used a new method, specially suited for this type of analysis [11]. This is a modified version of the one of Campbell, based on grouping the coefficients in sets with a well defined $\Delta \to 0$ limit.

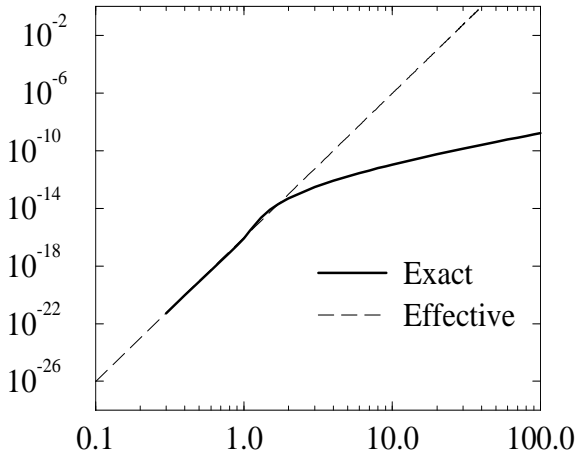**Figure 3**: $\gamma\nu \to \gamma\gamma\nu$ cross section in fb as a function of ω/m_e.

The keypoints of the method are mainly two:

(1) The use of a specific representation for the polarization vectors.

(2) By means of γ-algebra and spinor manipulations, we can reconstruct in the numerators the structures that appear in the denominators rather than making a tensorial decomposition. In that way all the results are expressed in terms of scalar functions with 3 and 4 denominators, rank-1 integrals with 3 and 4 denominators, rank-2 integrals with 3 denominators, and rank-3 functions with 3 denominators. This already provides an important simplification with respect to the standard decomposition, in that the computation of tensors such as

$$T^{\mu\nu;\,\mu\nu\rho;\,\mu\nu\rho\sigma} = \int d^n q \, \frac{q^\mu q^\nu;\; q^\mu q^\nu q^\rho;\; q^\mu q^\nu q^\rho q^\sigma}{D_0 \, D_{-1} \, D_2 \, D_{(23)}} \tag{8}$$

is completely avoided.

We made a large use of the Kahane–Chisholm manipulations over γ matrices [12]. Such identities are strictly four-dimensional, while we are, at the same time, using dimensional regularization. Our solution [8] is splitting, before any trace manipulation, the n-dimensional integration momentum appearing in the traces as [11] $q \to q + \tilde{q}$, where q and $\tilde{q}$ are the four-dimensional and ϵ-dimensional components ($\epsilon = n - 4$), respectively, so that $q \cdot \tilde{q} = 0$. The net effects are, on the one hand, that the γ algebra can then be safely performed in four dimensions and, on the other hand, that a set of extra integrals containing powers of $\tilde{q}^2$ in the numerator arise, but they are straightforward to compute.

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As an example, we have plotted in Fig. 3 the result of the direct computation for the cross section of the second process [8] in Eq. (2). The result is in full agreement with those reported in [9]. From the plot it is also clear that the effective theory is valid only when $\omega \leq 2m_e$, as expected. Since the exact formulae are too involved to be given explicitly, we followed two approaches. We first tried to extend the validity of the effective theory by computing the next-to-leading term. While this was a nice and completely independent check of the results of the effective theory, we found that it is not sufficient to enlarge the range of validity. A second approach was to fit our curves. For instance, $\sigma(\gamma\nu \to \gamma\gamma\nu) = \sigma^{\text{eff}}(\gamma\nu \to \gamma\gamma\nu) \times r^{-2.76046} \times \exp\left[2.13317 - 2.12629 \log^2(r) + 0.406718 \log^3(r) - 0.029852 \log^4(r)\right]$, where the effective cross section σ^{eff} is given in Eq. (26) of [8]. The range of validity is now between $1.7 \leq r = \omega/m_e \leq 100$. These fits are useful when these results are used for simulations in supernovae dynamics. Finally, in [13] we studied the leading contribution to the process $\gamma\nu \to \gamma\gamma\nu$ in supersymmetry with R-parity breaking and in left–right symmetric models (LRSM). The observations (i) and (ii) in section 2, also apply here. The leading additional contribution will come in a LRSM through the substitution of the $W^{\pm}$ and Z propagators by the corresponding $W'^{\pm}$ and Z', while in the supersymmetric case it will come through the substitution of the $W^{\pm}$ by a slepton. Indeed, it is interesting to notice that in the second case the lepton number is no longer conserved, i.e. transitions into different neutrino species are allowed. In particular, we found that in this process a muon-neutrino prefers to convert into a tau-neutrino rather than an electron-neutrino, in agreement with SuperKamiokande results. However, our processes hold for energies much below the energy of the atmospheric neutrinos (> 1 GeV) and the cross sections are still too small. We found that the cross sections in the $\not{R}_p$ MSSM are enhanced by a factor of the order of few 10% at best, relative to the SM cross sections, while the correction in LRSM models is negligible.

IV. ASTROPHYSICAL IMPLICATIONS

While the process $\gamma\gamma \to \gamma\nu\bar{\nu}$ provides an energy loss mechanism for stellar process and, in particular, it could be important in the cooling of neutron stars, the other two processes $\nu\gamma \to \nu\gamma\gamma$ and $\nu\bar{\nu} \to \gamma\gamma\gamma$ affect the mean free path of a neutrino inside a supernova ans should be included in the supernova codes.

In [14] it was found, using a Monte Carlo and the results of the effective theory [4,5] that for a large range of values of the temperature (T) and chemical potential (μ) the mean free path was less than the size of a supernova core (10^6 cm). This result remains valid when using the exact computation for certain values of μ and T and at energies not to far from the e^+e^- pair production. However, being the exacts results available it would be of extreme interest to find out the precise values.

Also in [14] it was predicted using the data of the supernova SN1987A that the exponent

of the energy dependence in the cross section $\sigma(\gamma\nu \to \gamma\gamma\nu) \propto \omega^\gamma$ should drop out from $\gamma = 10$ to less than 8.4 for ω a few MeV. Using our results we confirmed this prediction and indeed, we found that the exponent drops around 3 for energies between 1 to 10 MeV.

Finally, concerning the possible cosmological implications it was suggested in [4] that the process $\gamma\nu \to \gamma\gamma\nu$ might be relevant for some cosmological considerations if the decoupling temperature [15] in the exact case is found to be low enough. With the exacts results we found [8] that the temperature is too high to be of any relevance in cosmology.

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